MAT multiple-choice preparation July 2023

Queries (and corrections where I've made a mistake) to Martin Thomas at martin@workersliberty.org

These examples are chosen to help you learn how to do MAT multiple-choice questions *without detailed calculation*, either from *a diagram* or from considering what *sort of answer* (not what exact answer) might be right. The skills involved here are not taught at all for A-level but are useful for MAT and for real-life and university maths. You can also check out the "official" solutions at

https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test

Those are usually much longer and more laborious, i.e. not as good. The official solutions for the other (not multiple-choice) questions tend to be ok, though not always the best by any means.

Best to do these exercises *in pairs, helping each other,* if you can. Please *write down your reasoning* (even though you don't have to for the multiple-choice questions in the actual MAT exam) so as to "teach yourself" (or your co-worker).

For each question: spend five minutes trying to get the answer.

Then, if you are baffled, look at the hints. Try again.

If you are *still* baffled, look at the official solution. If you and your co-worker at still baffled (that'll happen), email me at martin@workersliberty.org.

You'll also find it useful to try the "pre-STEP" problems I gave you. If you can get through all those, then try some complete MAT papers. Give yourself 2.5 hours, see what you can manage, check with the official solutions. You will almost certainly *do badly at first*. Then try another complete paper. Then go back to the first complete paper, and try it again, using your memory of the official solutions.

MAT 2021 1 A

A. A regular dodecagon is a 12-sided polygon with all sides the same length and all internal angles equal. If I construct a regular dodecagon by connecting 12 equally-spaced points on a circle of radius 1, then the area of this polygon is

(a) $6+3\sqrt{3}$, (b) $2\sqrt{2}$, (c) $3\sqrt{2}$, (d) $3\sqrt{3}$, (e) 3.

MAT 2021 1 D

D. The area of the region bounded by the curve $y = e^x$, the curve $y = 1 - e^x$, and the y-axis equals

(a) 0, (b) $1 - \ln 2$, (c) $\frac{1}{2} - \frac{1}{2}\ln 2$,
(d) $\ln 2 - 1$, (e) $1 - \ln \frac{1}{2}$.

[Note that $\ln x$ is alternative notation for $\log_e x$.]

Extra help: ln 2 is about 0.69. (Worth memorising this).

MAT 2021 1 J

J. Four distinct real numbers a, b, c, and d are used to define four points

$$A = (a, b), \quad B = (b, c), \quad C = (c, d), \quad D = (d, a).$$

The quadrilateral $ABCD$ has all four sides the same length

(a) if and only if $(a - b)^2 = (c - d)^2$,

(b) if and only if $(a - c)^2 = (b - d)^2$,

(c) if and only if $(a - d)^2 = (b - c)^2$,

(d) if and only if $a - b + c - d = 0$,

(e) for no values of a, b, c, d.

Hints

MAT 2021 1 A

Hint

Draw a rough diagram.

The area of the circle is what?

The area of the polygon will be… what? Much bigger than the area of the circle? Only a little bigger than the area of the circle? Only a little smaller than the area of the circle? Much smaller than the area of the circle?

Which of the options fits your estimate?

Alternative hint

Divide the dodecagon into the 12 triangles. Work out the area of each triangle. Multiply by 12. (That is pretty much the official solution).

MAT 2021 1 D

Hint

Draw a rough diagram.

Shade the area mentioned.

It looks a bit like an isosceles triangle, but with "sides" not straight lines but curving inwards.

So its area is a bit less than an isosceles triangle with the same base and height.

The base is…. The height is…. So the area is a bit less than…..

Which of the options given fits that?

MAT 2021 1 F

Draw a rough diagram of the cubic, and mark the point (2,0).

Run a ruler along the curve so it's roughly a tangent to it at each point. How many times does the ruler go through (2,0)?

MAT 2021 1 J

Hint

You can rule out the "no values of a, b, c, d" option by seeing that ABCD can have four sides the same length for (0,0), (0,1), (1,1), (1,0).

If any value of (a, b, c, d) works, then e.g. (2a+3, 2b+3, 2c+3, 2d+3) (and put any numbers in place of 2 and 3) must work too. It enlarges ABCD by a factor of 2 (or whatever), and moves it by a distance of 3 (or whatever) both up and across.

Also, if any value of (a, b, c, d) works, then shuffling the numbers along, e.g. (b, c, d, a), (c, d, a, b), (d, a, b, c) must work too, because it only relabels the vertices of the quadrilateral and doesn't change its shape.

So the condition on a, b, c, d must be a *linear* one (no squares or such), and one which still works if you e.g. swap b for a, c for b, d for c, a for d.

So only one of the options is even *the right sort* of condition.

Alternative hint

ABCD is a rhombus, so has sides parallel as well as equal.

Draw a diagram.

x-shift moving from A to B equals x-shift moving from D to C

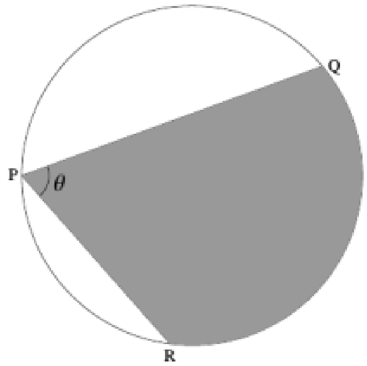
y-shift moving from A to B equals y-shift moving from D to C…

1. MAT 2011, B. A rectangle has perimeter P and area A. The values P and A must satisfy:

(a) $P^3 > A$, (b) $A^2 > 2P + 1$, (c) $P^2 \geq 16A$, (d) $PA \geq A+P$.

2. MAT 2012, J

J. If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram below,



then the largest possible area of the shaded region RPQ is

(a) $\theta\left(1+\cos\left(\frac{\theta}{2}\right)\right)$; (b) $\theta+\sin\theta$; (c) $\frac{\pi}{2}(1-\cos\theta)$; (d) θ.

3. MAT 2010, B

B. The sum of the first $2n$ terms of

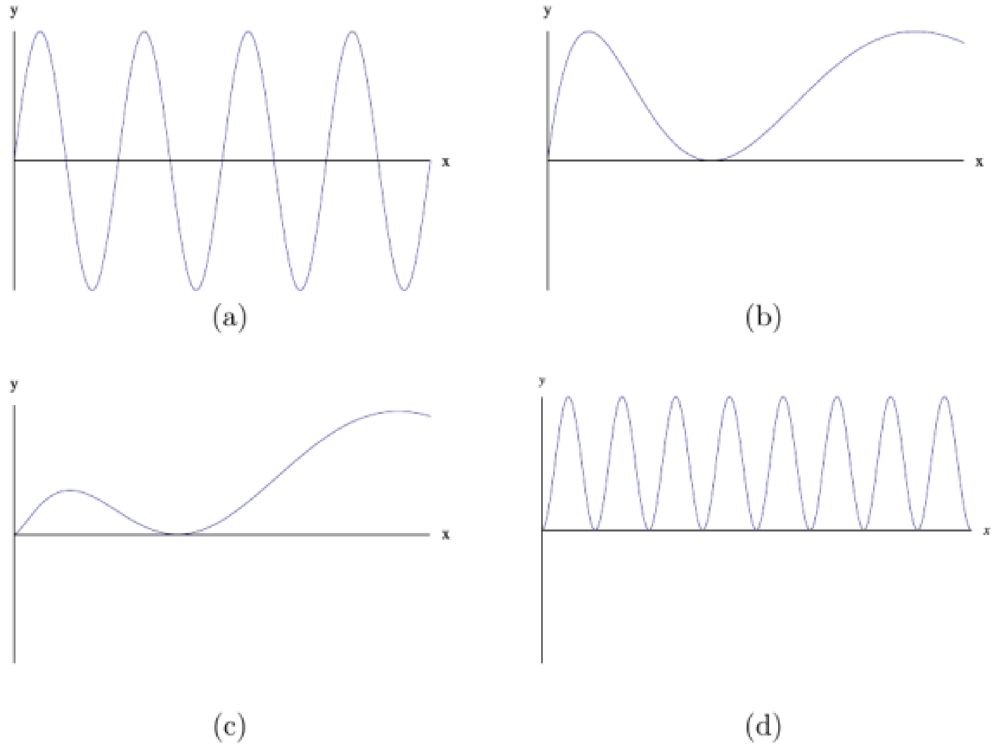
$$1, 1, 2, \frac{1}{2}, 4, \frac{1}{4}, 8, \frac{1}{8}, 16, \frac{1}{16}, \ldots$$

is

(a) $2^n + 1 - 2^{1-n}$, (b) $2^n + 2^{-n}$, (c) $2^{2n} - 2^{3-2n}$, (d) $\frac{2^n - 2^{-n}}{3}$.

4. MAT 2010, D

D. The graph of $y = \sin^2\sqrt{x}$ is drawn in



5. MAT, 2009, D

D. The smallest positive integer n such that

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + (-1)^{n+1}\, n \geqslant 100,$$

is

(a) 99, (b) 101, (c) 199, (d) 300.

6. (not a MAT question, but same sort of thing)
Without using a calculator, find which is bigger of 99^{100} and 100^{99}.

7. (a STEP problem: STEP 1 1999, Q.1)

How many integers greater than or equal to zero and less than a million are not divisible by 2 or 5? What is the average value of these integers?

How many integers greater than or equal to zero and less than 4179 are not divisible by 3 or 7? What is the average value of these integers?

This entry was posted in STEP and MAT on June 17, 2015 [https://mathsmartinthomas.wordpress.com/2015/06/17/using-simple-special-cases-in-problem-solving/] .

A minimum of blind calculation

"All limits, especially national ones, are contrary to the nature of mathematics… Mathematics knows no races… For mathematics the whole cultural world is a single country" – David Hilbert. "Face problems with a minimum of blind calculation, a maximum of seeing thought" – Hermann Minkowski

Hints for “MAT problems using simple special cases”

Hints for the problems at “Using simple special cases in problem-solving”.

1. Suppose it’s a 1×1 square. Which of the inequalities is valid? And suppose it is a very tiny square (say side 0.0001). Which of the inequalities is valid?

You can also do this by symmetry-type arguments. For any given shape of rectangle, A increases with the square of P. So the “shape” of the answer must be an equation connecting what power of P with what power of A?

Or another symmetry-type argument. The greatest rectangle area for any given perimeter P is got by a square. (Generally, the greatest polygon area for any given perimeter is got by a regular polygon, and the greatest area overall for any given perimeter is got by a circle). So A≤(the area of a square with perimeter P.

2. Think about the maximum area when $\theta=\pi/2$, and when θ is very small.

3. Think about n=1.

4. Can y be negative? What value are all its maxima? Are its maxima equally spaced?

5. Never mind about 100 for the moment. Find n for when the sum≥2, then n for when the sum≥3, then n for when the sum ≥4. See the pattern…

6. Never mind about 99 and 100 for the present. Compare:

1^2 and 2^1

2^3 and 3^2

3^4 and 4^3

See the trend. If you want to be sure the trend is real, work out:

$$\frac{n^{n+1}/(n+1)^n}{(n-1)^n/n^{(n-1)}}$$

7. Never mind about 1,000,000 for the moment. Work out the 2-and-5 problem for $0 \le n < 10$, and the 3-and-7 problem for $0 \le, n < 21$. See if you can build on that.

This entry was posted in STEP and MAT on April 17, 2015 [https://mathsmartinthomas.wordpress.com/2015/04/17/hints-for-mat-problems-using-simple-special-cases/] .