Pre-STEP problems

City of London Academy Southwark

Session 1

1. Into a maximum of how many regions do n lines divide a plane?

2. What is the length of the shortest route for an ant from a vertex of a cube to the opposite vertex?

3. Ashley throws a ball aiming directly at Brenda, who is perched in a tree. At exactly the same time, Brenda falls from the tree. Can she catch the ball?

Session 2

1. What are the square roots of i?

2. Prove $n^3 - n$ is divisible by 6 for all n

3. The line $y = ax + b$, with $a \neq 0$, can be upward-sloping and downward-sloping. Erase the axes and stretch the paper one way or another, and any two upward-sloping line can be equated to each other, or any two downward-sloping ones. So there are two basic shapes for the line. Sketch the basic shapes for the quadratic curve $y = ax^2 + bx + c$ ($a \neq 0$). Sketch the basic shapes for the cubic curve $y = ax^3 + bx^2 + cx + d$ ($a \neq 0$).

Session 3

1. $c^2 = a^2 + b^2$ and a, b, c are all whole numbers. Prove that they can't all be odd (in other words, at least one must be even). Then prove that at least one of a and b must be even. Then prove that at least one of a and b must be divisible by 4

2. If I had a cube and six colours and painted each face a different colour, how many (different) ways could I paint the cube? What about if I had n colours instead of 6?

3. Sketch the graph of $y = \cos \frac{1}{x}$.