5 Methods: Symmetry and simple-cases

Without calculating $(a+b)^3$, you know that the coefficient of a^2b in the answer must be equal to the coefficient of ab^2. You could swap a and b in the question without making a difference, so you must be able to swap a and b in the answer without making a difference.

If the coefficient of a^3b^2 in $(a+b)^5$ is 10, what is the coefficient of a^2b^3?

Seeing a **symmetry** in the question which means that there must be a similar symmetry in the answer often helps.

At the very least, you can avoid duplication of working by seeing symmetry in the question.

If you want to know what all the coefficients of all the terms in $(a+b)^5$ add up to, you don't have to work out all the terms. Considering one special **simple case** will give you the answer. The coefficients must still be valid if $a = 1, b = 1$, when all the powers of a and b are also 1. So what do the coefficients add up to?

Another way you can use **simple cases** in many problems is first to consider a *simplified* version of the problem, and then to build the full solution from that.

5.1 MAT problems which can be done just from symmetry or special-cases

Some solutions to these in appendix to this booklet.

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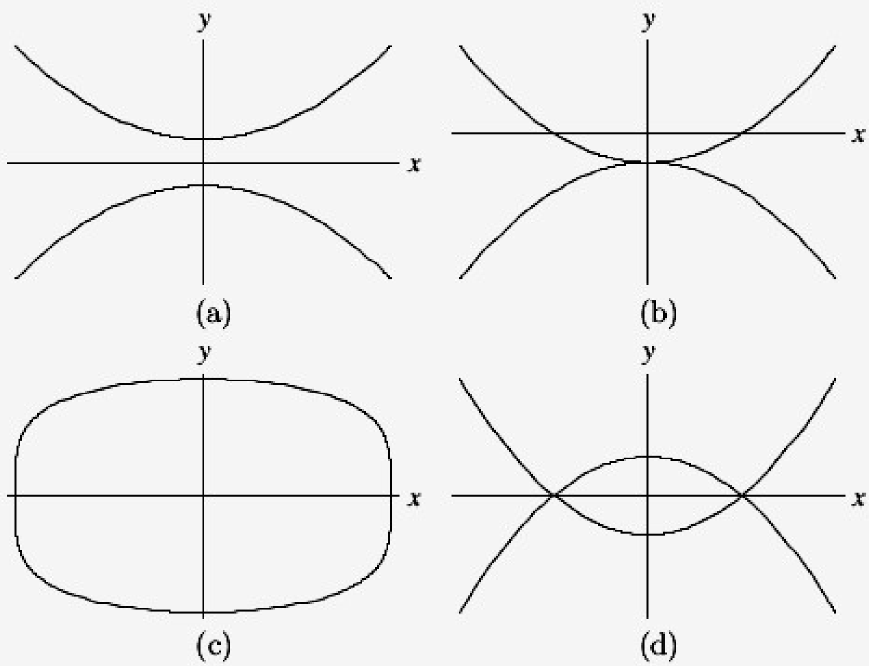
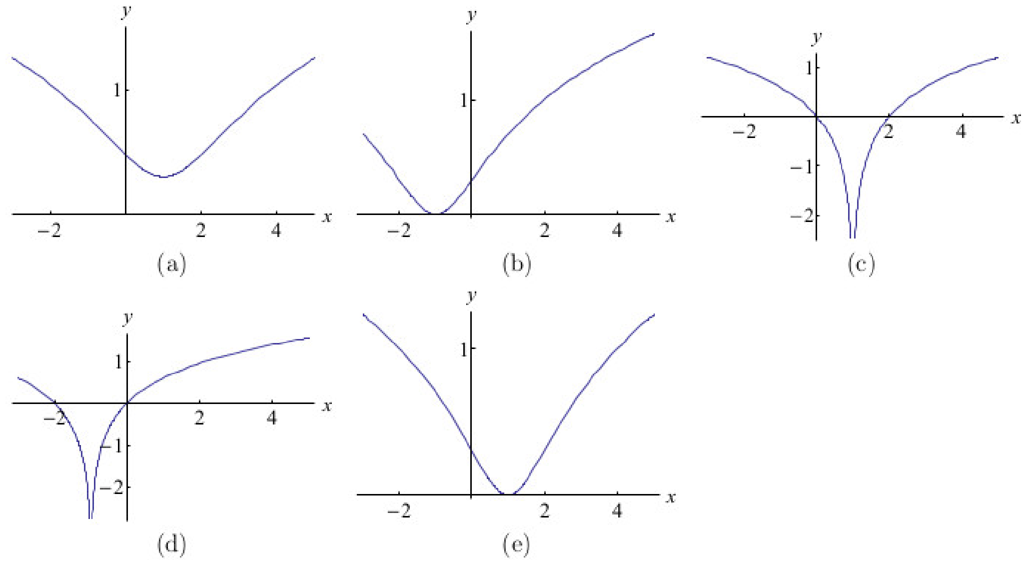
1. MAT 2013 d

D. Which of the following sketches is a graph of $x^4 - y^2 = 2y + 1$?



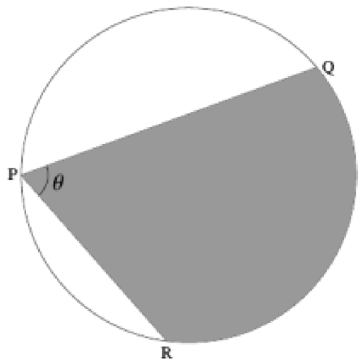
2. MAT 2014 b

B. The graph of the function $y = \log_{10}(x^2 - 2x + 2)$ is sketched in



3. MAT 2012 j

J. If two chords QP and RP on a circle of radius 1 meet in an angle θ at P, for example as drawn in the diagram below,



then the largest possible area of the shaded region RPQ is

(a) $\theta\left(1+\cos\left(\frac{\theta}{2}\right)\right)$; (b) $\theta+\sin\theta$; (c) $\frac{\pi}{2}(1-\cos\theta)$; (d) θ.

4. MAT 2014 d

D. The reflection of the point $(1,0)$ in the line $y = mx$ has coordinates

(a) $\left(\frac{m^2+1}{m^2-1}, \frac{m}{m^2-1}\right)$, (b) $(1,m)$, (c) $(1-m,m)$,

(d) $\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$, (e) $\left(1-m^2,m\right)$.

5. MAT 2014 f - The functions S and T are defined for real numbers by:

$S(x) = x + 1$; and $T(x) = -x$

The function S is applied s times and the function T is applied t times, in some order, to produce the function $F(x) = 8 - x$

It is possible to deduce that: (a) s = 8 and t = 1. (b) s is odd and t is even. (c) s is even and t is odd. (d) s and t are powers of 2. (e) none of the above.

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6. MAT 2010 b

B. The sum of the first $2n$ terms of

$$1, 1, 2, \frac{1}{2}, 4, \frac{1}{4}, 8, \frac{1}{8}, 16, \frac{1}{16}, \ldots$$

is

(a) $2^n + 1 - 2^{1-n}$, (b) $2^n + 2^{-n}$, (c) $2^{2n} - 2^{3-2n}$, (d) $\dfrac{2^n - 2^{-n}}{3}$.

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7. MAT 2014 h

H. The function $F(n)$ is defined for all positive integers as follows: $F(1) = 0$ and for all $n \geqslant 2$,

$$\begin{aligned} F(n) &= F(n-1) + 2 && \text{if 2 divides } n \text{ but 3 does not divide } n; \\ F(n) &= F(n-1) + 3 && \text{if 3 divides } n \text{ but 2 does not divide } n; \\ F(n) &= F(n-1) + 4 && \text{if 2 and 3 both divide } n; \\ F(n) &= F(n-1) && \text{if neither 2 nor 3 divides } n. \end{aligned}$$

The value of $F(6000)$ equals

(a) 9827, (b) 10121, (c) 11000, (d) 12300, (e) 12352.

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8. MAT 2014 j

J. For all real numbers x, the function $f(x)$ satisfies

$$6 + f(x) = 2f(-x) + 3x^2 \left(\int_{-1}^{1} f(t)\,\mathrm{d}t \right).$$

It follows that $\int_{-1}^{1} f(x)\,\mathrm{d}x$ equals

(a) 4, (b) 6, (c) 11, (d) $\dfrac{27}{2}$, (e) 23.

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9. MAT 2013 h - The area bounded by the graphs $y = \sqrt{(2 - x^2)}$ and $x + (\sqrt{2} - 1)y = \sqrt{2}$ equals: (a) $\frac{\sin\sqrt{2}}{\sqrt{2}}$; (b) $\frac{\pi}{4} - \frac{1}{\sqrt{2}}$; (c) $\frac{\pi}{2\sqrt{2}}$; (d) $\pi^2 - 6$.

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10. MAT 2013 i - The function F(k) is defined for positive integers by $F(1) = 1, F(2) = 1, F(3) = -1$ and by the identities $F(2k) = F(k), F(2k+1) = F(k)$, for $k \geq 2$. The sum $F(1) + F(2) + F(3) + \ldots + F(100)$ equals (a) -15; (b) 28; (c) 64; (d) 81

5.2 I/1999/1

I/1999/1

How many integers greater than or equal to zero and less than a million are not divisible by 2 or 5? What is the average value of these integers?

How many integers greater than or equal to zero and less than 4179 are not divisible by 3 or 7? What is the average value of these integers?