REQUIRED FORMULAE

Roots of polynomials		
Formula	**Comment**	**Papers**
$ax^2 + bx + c = 0$ has roots $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		1, 2, 3
For $ax^2 + bx + c = 0$ with roots α and β: $\alpha + \beta = -b/a, \ \alpha\beta = c/a$		1, 2, 3
For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ: $\alpha + \beta + \gamma = -b/a, \ \alpha\beta + \beta\gamma + \gamma\alpha = c/a, \ \alpha\beta\gamma = -d/a$	The pattern is the same for polynomial equations of higher degree	2, 3

Laws of indices		
Formula	**Comment**	**Papers**
$a^x a^y = a^{x+y}$		1, 2, 3
$a^0 = 1$	$a \neq 0$	1, 2, 3
$(a^x)^y = a^{xy}$		1, 2, 3
$a^x = e^{x \ln a}$	defines a^x when x is not an integer	1, 2, 3 ●

Laws of logarithms		
Formula	**Comment**	**Papers**
$x = a^n \Leftrightarrow n = \log_a x$	$x > 0,\ a > 0\ (a \neq 1)$	1, 2, 3
$\log_a x + \log_a y = \log_a(xy)$		1, 2, 3
$\log_a x - \log_a y = \log_a(x/y)$		1, 2, 3
$k \log_a x = \log_a x^k$	for $x > 0$	1, 2, 3

Sequences and series		
Formula	**Comment**	**Papers**
General (nth) term of an arithmetic progression: $u_n = a + (n-1)d$	d is the common difference	1, 2, 3
General (nth) term of a geometric progression: $u_n = ar^{n-1}$	r is the common ratio	1, 2, 3
Sum of an arithmetic progression: $S_n = \frac{1}{2}n\{2a + (n-1)d\}$	or: $S_n = an + \frac{1}{2}n(n-1)d$	1, 2, 3 ●
Sum of a geometric progression: $S_n = \dfrac{a(1-r^n)}{1-r}$		1, 2, 3 ●
Sum to infinity of a geometric progression: $S_\infty = \dfrac{a}{1-r}$	$\|r\| < 1$	1, 2, 3 ●
$^nC_r = \dfrac{n!}{(n-r)!\,r!}$		1, 2, 3 ●
$(a+b)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$	Binomial expansion, $n \in \mathbb{N}$	1, 2, 3 ●
$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \cdots + \frac{k(k-1)\cdots(k-r+1)}{r!}x^r + \cdots$	$\|x\| < 1, \; k \in \mathbb{Q}$	1, 2, 3 ●
$\displaystyle\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$		1, 2, 3 ●
$\mathrm{f}(x) = \displaystyle\sum_{r=0}^{\infty} \frac{1}{r!}\,\mathrm{f}^{(r)}(0)\,x^r$	Maclaurin series	3 ●
$\mathrm{e}^x = \displaystyle\sum_{r=0}^{\infty} \frac{x^r}{r!}$	converges for all x	2, 3 ●
$\ln(1+x) = \displaystyle\sum_{r=1}^{\infty} (-1)^{r+1}\frac{x^r}{r}$	converges for $-1 < x \leqslant 1$	3 ●
$\sin x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r+1}}{(2r+1)!}$	converges for all x	3 ●
$\cos x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r}}{(2r)!}$	converges for all x	3 ●

Coordinate geometry		
Formula	**Comment**	**Papers**
The straight line graph with gradient m passing through the point (x_1, y_1) has equation $y - y_1 = m(x - x_1)$		1, 2, 3
Straight lines with non-zero gradients m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$		1, 2, 3

Trigonometry		
Formula	**Comment**	**Papers**
Sine rule for the triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$		1, 2, 3
Cosine rule in the triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos A$		1, 2, 3
Area of triangle ABC: $\frac{1}{2}ab \sin C$		1, 2, 3
$\cos^2 A + \sin^2 A = 1$		1, 2, 3
$\sec^2 A = 1 + \tan^2 A$		1, 2, 3
$\operatorname{cosec}^2 A = 1 + \cot^2 A$		1, 2, 3
$\sin 2A = 2 \sin A \cos A$		1, 2, 3
$\cos 2A = \cos^2 A - \sin^2 A$		1, 2, 3
$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$	$A \neq (k + \frac{1}{2})\frac{\pi}{2}$, $k \in \mathbb{Z}$	1, 2, 3
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$		1, 2, 3
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$		1, 2, 3
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$A \pm B \neq (k + \frac{1}{2})\pi$, $k \in \mathbb{Z}$	1, 2, 3 ●
$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$	θ small (compared with 1); θ in radians	1, 2, 3 ●

Hyperbolic functions		
Formula	**Comment**	**Papers**
$\sinh x = \dfrac{e^x - e^{-x}}{2}$	by definition	3
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	by definition	3
$\tanh x = \dfrac{\sinh x}{\cosh x}$	by definition	3
$\cosh^2 A - \sinh^2 A = 1$		3 ●
$\text{sech}^2 A = 1 - \tanh^2 A$		3 ●
$\text{cosech}^2 A = \coth^2 A - 1$		3 ●
$\sinh 2A = 2\sinh A \cosh A$		3 ●
$\cosh 2A = \cosh^2 A + \sinh^2 A$		3 ●
$\tanh 2A = \dfrac{2\tanh A}{1 + \tanh^2 A}$		3 ●
$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$		3 ●
$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$		3 ●
$\tanh(A \pm B) = \dfrac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$		3 ●

Derivatives			
Function	**Derivative**	**Comment**	**Papers**
$\sin x$	$\cos x$		1, 2, 3
$\cos x$	$-\sin x$		1, 2, 3
$\tan x$	$\sec^2 x$		1, 2, 3 ●
$\cot x$	$-\mathrm{cosec}^2 x$		1, 2, 3 ●
$\sec x$	$\sec x \tan x$		1, 2, 3 ●
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$		1, 2, 3 ●
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$		2, 3 ●
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$		2, 3 ●
$\sinh x$	$\cosh x$		3
$\cosh x$	$\sinh x$		3
$\tanh x$	$\mathrm{sech}^2 x$		3 ●
$\coth x$	$-\mathrm{cosech}^2 x$		3 ●
$\mathrm{sech}\, x$	$-\mathrm{sech}\, x \tanh x$		3 ●
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$		3 ●
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$		3 ●
e^x	e^x		1, 2, 3
$\ln x$	$\dfrac{1}{x}$		1, 2, 3
$\mathrm{f}(x) + \mathrm{g}(x)$	$\mathrm{f}'(x) + \mathrm{g}'(x)$		1, 2, 3
$\mathrm{f}(x)\mathrm{g}(x)$	$\mathrm{f}'(x)\mathrm{g}(x) + \mathrm{f}(x)\mathrm{g}'(x)$	product rule	1, 2, 3
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\big(\mathrm{g}(x)\big)^2}$	quotient rule	1, 2, 3 ●
$\mathrm{f}\big(\mathrm{g}(x)\big)$	$\mathrm{f}'\big(\mathrm{g}(x)\big)\mathrm{g}'(x)$	chain rule	1, 2, 3

Integrals			
Function	**Integral**	**Comment**	**Papers**
x^n	$\frac{1}{n+1} x^{n+1} + c$	$n \neq -1$	1, 2, 3
x^{-1}	$\ln|x| + c$		1, 2, 3
$\cos x$	$\sin x + c$		1, 2, 3
$\sin x$	$-\cos x + c$		1, 2, 3
$\sinh x$	$\cosh x + c$		3
$\cosh x$	$\sinh x + c$		3
$\dfrac{1}{\sqrt{1 - x^2}}$	$\sin^{-1} x + c$	$-1 < x < 1$	2, 3 ●
$\dfrac{1}{1 + x^2}$	$\tan^{-1} x + c$		2, 3 ●
e^x	$e^x + c$		1, 2, 3
$f'(x) + g'(x)$	$f(x) + g(x) + c$		1, 2, 3
$f'\big(g(x)\big) g'(x)$	$f\big(g(x)\big) + c$		1, 2, 3
$\dfrac{f'(x)}{f(x)}$	$\ln|f(x)| + c$		1, 2, 3 ●
$\big(f(x)\big)^n f'(x)$	$\frac{1}{n+1} \big(f(x)\big)^{n+1} + c$	$n \neq -1$	1, 2, 3 ●
$u\dfrac{\mathrm{d}v}{\mathrm{d}x}$	$uv - \displaystyle\int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$	integration by parts	1, 2, 3 ●

General calculus		
Formula	**Comment**	**Papers**
$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	first principles definition	1, 2, 3 ●
$\dfrac{dy}{dx} = \dfrac{dy}{dt} \bigg/ \dfrac{dx}{dt}$	for the parameterised curve $y = y(t)$, $x = x(t)$	1, 2, 3 ●
Area under the curve $y = f(x)$ and above the x-axis: $\displaystyle\int_a^b f(x)\,dx$		1, 2, 3
Volume of revolution about the x-axis: $\pi \displaystyle\int_a^b \left(f(x)\right)^2 dx$		3
$\displaystyle\int_a^b y\,dx \approx \tfrac{1}{2}h(y_0 + y_n) + h(y_1 + y_2 + \cdots + y_{n-1})$	$h = \dfrac{b-a}{n}$, $y_r = y(a + rh)$, trapezium rule	1, 2, 3 ●
$\ddot{x} = -\omega^2 x \Rightarrow x = R\sin(\omega t + \alpha)$ or $\quad x = R\cos(\omega t + \beta) \quad$ or $\quad x = A\cos\omega t + B\sin\omega t$	simple harmonic motion	3 ●

Circles		
Formula	**Comment**	**Papers**
Length of an arc of a circle of radius r: $r\theta$	θ is angle subtended in radians	1, 2, 3
Area of a sector of a circle of radius r: $\frac{1}{2}r^2\theta$	θ is angle subtended in radians	1, 2, 3

Complex numbers		
Formula	**Comment**	**Papers**
$e^{i\theta} = \cos\theta + i\sin\theta$		3
$z = r(\cos\theta + i\sin\theta) \Rightarrow z^n = r^n(\cos n\theta + i\sin n\theta)$	de Moivre's theorem	3 •
$z^n = 1$ has roots $z = e^{2\pi ki/n}$ where $k = 0, 1, \ldots, (n-1)$	Roots of unity	3 •
Half line with end-point a: $\arg(z - a) = \theta$	θ is the angle between the line and a line parallel to the positive real axis	2, 3
Circle, centre a and radius r: $|z - a| = r$		2, 3

Vectors		
Formula	**Comment**	**Papers**
$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$		1, 2, 3
$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}|\,|\mathbf{b}|\cos\theta$	scalar product, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$	2, 3
$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$	vector product	3 •
$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}||\sin\theta|$	θ is the acute angle between the vectors	3 •
Equation of the line through the point with position vector $\mathbf{a}$ parallel to $\mathbf{b}$: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$		2, 3
Equation of the plane containing the point with position vector $\mathbf{a}$ and with normal $\mathbf{n}$: $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$		3

Matrices		
Formula	**Comment**	**Papers**
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det \mathbf{A} = ad - bc$		2, 3
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{A}^{-1} = \dfrac{1}{\det \mathbf{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	$\det \mathbf{A} \neq 0$	2, 3
$\mathbf{AB}$ is equivalent to $\mathbf{B}$ then $\mathbf{A}$	for transformations represented by these matrices	2, 3
$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	$\det \mathbf{AB} \neq 0$	2, 3
$\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$	reflection in the line $y = \pm x$	2, 3 ●
$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$	rotation by θ about the z-axis; the direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation	2, 3 ●
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	reflection in the plane $z = 0$	2, 3 ●

Mechanics		
Formula	**Comment**	**Papers**
mg	weight	1, 2, 3
$F \leqslant \mu R$	frictional force related to normal reaction R	1, 2, 3
$F = ma$	scalar version of Newton's second law; constant mass	1, 2, 3
$\mathbf{F} = m\mathbf{a}$	vector version of Newton's second law; constant mass	1, 2, 3
$\frac{1}{2}mv^2$	kinetic energy	2, 3 ●
mgh	change in gravitational potential energy; h is vertical height	2, 3 ●
mv	momentum	2, 3 ●
$mv - mu$	impulse	2, 3 ●
$T = \dfrac{\lambda x}{l} = kx$	Hooke's law	2, 3 ●
$E = \dfrac{\lambda x^2}{2l} = \dfrac{1}{2}kx^2$	elastic potential energy	2, 3 ●
$v = \dfrac{\mathrm{d}r}{\mathrm{d}t}, \quad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2r}{\mathrm{d}t^2}$	motion in a straight line (where acceleration, a, may not be constant)	1, 2, 3 ●
$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad s = \frac{1}{2}(u+v)t, \quad v^2 - u^2 = 2as$	motion in a straight line with constant acceleration, a	1, 2, 3 ●
$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \quad \mathbf{a} = \dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	motion in two (STEP 1) or three (STEP 3) dimensions where acceleration, $\mathbf{a}$, may not be constant)	1, 2, 3 ●
$\mathbf{v} = \mathbf{u} + \mathbf{a}t, \quad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2, \quad \mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t, \quad \mathbf{v}.\mathbf{v} - \mathbf{u}.\mathbf{u} = 2\mathbf{a}.\mathbf{s}$	motion in two (STEP 1) or three (STEP 2) dimensions with constant acceleration, $\mathbf{a}$	1, 2, 3 ●
$v_1 - v_2 = -e(u_1 - u_2)$ or relative speed of separation $= e \times$ relative speed of approach	Newton's experimental law	2, 3 ●
speed $= r\dot{\theta}$, radial acceleration $= \dfrac{v^2}{r} = r\dot{\theta}^2$ towards the centre, tangential acceleration $= r\ddot{\theta}$	motion in a circle of radius r	3 ●

Probability/Statistics		
Formula	**Comment**	**Papers**
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	probability of the union of two events	1, 2, 3 ●
$P(A \cap B) = P(A \mid B)\,P(B)$	probability of the intersection of two events	1, 2, 3 ●
$E(aX + bY + c) = aE(X) + bE(Y) + c$	algebra of expectation	3 ●
$\operatorname{Var}(aX + bY + c) = a^2\operatorname{Var}(X) + b^2\operatorname{Var}(Y)$	algebra of variances for independent variables	3 ●
$\mu = E(X) = \sum_i x_i P(X = x_i)$	expectation of a discrete random variable X	1, 2, 3 ●
$\mu = E(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$	expectation of a continuous random variable X with p.d.f. f	2, 3 ●
$\sigma^2 = \operatorname{Var}(X) = \sum_i (x_i - \mu)^2 P(X = x_i)$ $= \sum_i x_i^2\,P(X = x_i) - \mu^2$	variance of a discrete random variable X	1, 2, 3 ●
$\sigma^2 = \operatorname{Var}(X) = \int (x - \mu)^2\mathrm{f}(x)\,\mathrm{d}x = \int x^2\mathrm{f}(x)\,\mathrm{d}x - \mu^2$	variance of a continuous random variable X with p.d.f. f	2, 3 ●
$\mathrm{F}(x) = P(X \leqslant x) = \int_{-\infty}^{x} \mathrm{f}(x)\,\mathrm{d}x$	cumulative distribution function (c.d.f.)	2, 3 ●

Random variables				
Distribution	P$(X = x)$	E(X)	Var (X)	**Papers**
Binomial B(n, p)	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	1, 2, 3 ●
Uniform distribution over 1, 2, ..., n	$\dfrac{1}{n}$	$\frac{1}{2}(n+1)$	$\frac{1}{12}(n^2 - 1)$ (included for completeness; memorisation not required)	1, 2, 3 ●
Poisson Po(λ)	$\dfrac{\lambda^x \mathrm{e}^{-x}}{x!}$	λ	λ	2, 3 ●
Distribution	p.d.f.	E(X)	Var (X)	**Papers**
Uniform distribution over $[a, b]$	$\dfrac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$ (included for completeness; memorisation not required)	2, 3 ●
Normal N(μ, σ^2)	$\dfrac{1}{\sqrt{2\pi\sigma^2}} \, \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ (included for completeness; memorisation not required)	μ	σ^2	1, 2, 3 ●