October 2016 FP2 test paper. 60 minutes, 55 marks

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1. Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\mathrm{i},$$

giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta \leq \pi$.

(6)

(Q.2 June 2009)

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2. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + \mathrm{i}}, \quad z \neq -\mathrm{i}.$$

The circle with equation $|z| = 3$ is mapped by T onto the curve C.

(a) Show that C is a circle and find its centre and radius. Do this *both* by the geometric method *and* by the algebraic method.

(8 for geometric)
(8 for algebraic)

The region $|z| < 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Shade the region R on an Argand diagram.

(2)

(Q.6 June 2009, adapted)4-5

(4)

3. Find the set of values of x for which

$$\frac{3}{x+3} > \frac{x-4}{x}.$$

(7)

(Q.1 June 2011, adapted)

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please turn over for Q.4-5

4. Given that

$$(2r + 1)^3 = Ar^3 + Br^2 + Cr + 1,$$

(*a*) find the values of the constants A, B and C.

(2)

(*b*) Show that

$$(2r + 1)^3 - (2r - 1)^3 = 24r^2 + 2.$$

(2)

(*c*) Using the result in part (*b*) and the method of differences, show that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n + 1)(2n + 1).$$

(5)

(Q.4 June 2011, adapted)

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5. (*a*) Use de Moivre's theorem to show that

$$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta.$$

(5)

Hence, given also that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$,

(*b*) find all the solutions of

$$\sin 5\theta = 5 \sin 3\theta,$$

in the interval $0 \leq \theta < 2\pi$. Give your answers to 3 decimal places.

(6)

(Q.6 June 2011, adapted)

Total 55 marks