Paper Reference(s)

## 6665/01

## Edexcel GCE

## Core Mathematics C3

Silver Level S4

## Time: 1 hour 30 minutes

## Materials required for examination papers <br> Mathematical Formulae (Green) Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 69 | 61 | 53 | 45 | 39 | 34 |

1. (a) Express $7 \cos x-24 \sin x$ in the form $R \cos (x+\alpha)$ where $R>0$ and $0<\alpha<\frac{\pi}{2}$.

Give the value of $\alpha$ to 3 decimal places.
(b) Hence write down the minimum value of $7 \cos x-24 \sin x$.
(c) Solve, for $0 \leq x<2 \pi$, the equation

$$
7 \cos x-24 \sin x=10
$$

giving your answers to 2 decimal places.
2. (a) Use the identity $\cos ^{2} \theta+\sin ^{2} \theta=1$ to prove that $\tan ^{2} \theta=\sec ^{2} \theta-1$.
(b) Solve, for $0 \leq \theta<360^{\circ}$, the equation

$$
\begin{equation*}
2 \tan ^{2} \theta+4 \sec \theta+\sec ^{2} \theta=2 \tag{6}
\end{equation*}
$$

June 2009
3. (a) Express $5 \cos x-3 \sin x$ in the form $R \cos (x+\alpha)$, where $R>0$ and $0<\alpha<\frac{1}{2} \pi$.
(b) Hence, or otherwise, solve the equation

$$
5 \cos x-3 \sin x=4
$$

for $0 \leq x<2 \pi$, giving your answers to 2 decimal places.

January 2010
4.

$$
\mathrm{f}(x)=\frac{2 x+2}{x^{2}-2 x-3}-\frac{x+1}{x-3} .
$$

(a) Express $\mathrm{f}(x)$ as a single fraction in its simplest form.
(b) Hence show that $\mathrm{f}^{\prime}(x)=\frac{2}{(x-3)^{2}}$.

January 2009
5. The mass, $m$ grams, of a leaf $t$ days after it has been picked from a tree is given by

$$
m=p \mathrm{e}^{-k t}
$$

where $k$ and $p$ are positive constants.
When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.
(a) Write down the value of $p$.
(b) Show that $k=\frac{1}{4} \ln 3$.
(4)
(c) Find the value of $t$ when $\frac{\mathrm{d} m}{\mathrm{~d} t}=-0.6 \ln 3$.
(6)

June 2011
6. (a) Prove that

$$
\begin{equation*}
\frac{1}{\sin 2 \theta}-\frac{\cos 2 \theta}{\sin 2 \theta}=\tan \theta, \quad \theta \neq 90 n^{\circ}, \quad n \in \mathbb{Z} \tag{4}
\end{equation*}
$$

(b) Hence, or otherwise,
(i) show that $\tan 15^{\circ}=2-\sqrt{3}$,
(ii) solve, for $0<x<360^{\circ}$,

$$
\operatorname{cosec} 4 x-\cot 4 x=1 .
$$

7. 

$$
\mathrm{h}(x)=\frac{2}{x+2}+\frac{4}{x^{2}+5}-\frac{18}{\left(x^{2}+5\right)(x+2)}, \quad x \geq 0
$$

(a) Show that $\mathrm{h}(x)=\frac{2 x}{x^{2}+5}$.
(b) Hence, or otherwise, find $\mathrm{h}^{\prime}(x)$ in its simplest form.


Figure 1
Figure 1 shows a graph of the curve with equation $y=\mathrm{h}(x)$.
(c) Calculate the range of $\mathrm{h}(x)$.

January 2013
8. The amount of a certain type of drug in the bloodstream $t$ hours after it has been taken is given by the formula

$$
x=D \mathrm{e}^{-\frac{-1}{8} t},
$$

where $x$ is the amount of the drug in the bloodstream in milligrams and $D$ is the dose given in milligrams.

A dose of 10 mg of the drug is given.
(a) Find the amount of the drug in the bloodstream 5 hours after the dose is given.

Give your answer in mg to 3 decimal places.

A second dose of 10 mg is given after 5 hours.
(b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

No more doses of the drug are given. At time $T$ hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg .
(c) Find the value of $T$.

June 2007

## END

\begin{tabular}{|c|c|c|}
\hline Question Number \& Scheme \& Marks \\
\hline 1. (a) \& \begin{tabular}{l}
\[
\begin{aligned}
\& 7 \cos x-24 \sin x=R \cos (x+\alpha) \\
\& 7 \cos x-24 \sin x=R \cos x \cos \alpha-R \sin x \sin \alpha \\
\& \text { Equate } \cos x: \quad 7=R \cos \alpha \\
\& \text { Equate } \sin x: \quad 24=R \sin \alpha
\end{aligned}
\]
\[
\begin{array}{lr}
R=\sqrt{7^{2}+24^{2}} ;=25 \& R=25 \\
\tan \alpha=\frac{24}{7} \Rightarrow \alpha=1.287002218 \ldots c \& \tan \alpha=\frac{24}{7} \text { or } \tan \alpha=\frac{7}{24} \\
\text { awrt } 1.287
\end{array}
\] \\
Hence, \(7 \cos x-24 \sin x=25 \cos (x+1.287)\)
\end{tabular} \& \begin{tabular}{l}
B1 \\
M1 \\
A1 \\
(3)
\end{tabular} \\
\hline (b) \& Minimum value \(=\underline{-25} \quad-25\) or \(-R\) \& \begin{tabular}{l}
B1ft \\
(1)
\end{tabular} \\
\hline (c) \&  \& M1
M1

M1
A1
A1
M)
[9] <br>
\hline
\end{tabular}



| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| 3. (a) | $5 \cos x-3 \sin x=R \cos (x+\alpha), \quad R>0,0<x<\frac{\pi}{2}$ |  |
|  | $5 \cos x-3 \sin x=R \cos x \cos \alpha-R \sin x \sin \alpha$ |  |
|  | Equate $\cos x$ : $5=R \cos \alpha$ |  |
|  | Equate $\sin x$ : $\quad 3=R \sin \alpha$ $R=\sqrt{5^{2}+3^{2}} ;=\sqrt{34}\{=5.83095 . .\}$ | $\begin{aligned} & \text { M1; } \\ & \text { A1 } \end{aligned}$ |
|  | $\tan \alpha=\frac{3}{5} \Rightarrow \alpha=0.5404195003 \ldots . .{ }^{\text {c }}$ | M1 |
|  |  | A1 |
|  | Hence, $5 \cos x-3 \sin x=\sqrt{34} \cos (x+0.5404)$ |  |
|  |  | (4) |
| (b) | $5 \cos x-3 \sin x=4$ |  |
|  | $\sqrt{34} \cos (x+0.5404)=4$ |  |
|  | $\cos (x+0.5404)=\frac{4}{\sqrt{34}}\{=0.68599 \ldots\}$ | M1 |
|  | $(x+0.5404)=0.814826916 \ldots{ }^{\text {c }}$ | M1 |
|  | $x=0.2744 . .{ }^{\text {c }}$ | A1 |
|  | $(x+0.5404)=2 \pi-0.814826916 . .{ }^{c}$ c $\left\{=5.468358 . . .{ }^{\text {c }}\right\}$ | M1 |
|  | $x=4.9279 \ldots{ }^{\text {c }}$ | A1 |
|  | Hence, $x=\{0.27,4.93\}$ |  |
|  |  | (5) |
|  |  | [9] |


| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| 4. (a) | $\begin{aligned} & \frac{2 x+2}{x^{2}-2 x-3}-\frac{x+1}{x-3}=\frac{2 x+2}{(x-3)(x+1)}-\frac{x+1}{x-3} \\ &=\frac{2 x+2-(x+1)(x+1)}{(x-3)(x+1)} \\ &=\frac{(x+1)(1-x)}{(x-3)(x+1)} \\ &=\frac{1-x}{x-3} \quad \text { Accept }-\frac{x-1}{x-3}, \frac{x-1}{3-x} \\ & \begin{aligned} \frac{\mathrm{d}}{\mathrm{~d} x}\left(\frac{1-x}{x-3}\right)= & \frac{(x-3)(-1)-(1-x) 1}{(x-3)^{2}} \\ & =\frac{-x+3-1+x}{(x-3)^{2}}=\frac{2}{(x-3)^{2}} \quad * \end{aligned} \quad \text { cso } \end{aligned}$ | M1 A1 <br> M1 <br> A1 (4) <br> M1 A1 <br> A1 (3) |
| 5. (a) ${ }^{\text {(b) }}$ | $\begin{aligned} & p=7.5 \\ & 2.5=7.5 e^{-4 k} \\ & e^{-4 k}=\frac{1}{3} \\ &-4 k=\ln \left(\frac{1}{3}\right) \\ &-4 k=-\ln (3) \\ & k=\frac{1}{4} \ln (3) \\ & \frac{\mathrm{d} m}{\mathrm{~d} t}=-k p e^{-k t} \\ &-\frac{1}{4} \ln 3 \times 7.5 e^{-\frac{1}{4}(\ln 3) t}=-0.6 \ln 3 \\ & e^{-\frac{1}{4}(\ln 3) t}=\frac{2.4}{7.5}=(0.32) \\ &-\frac{1}{4}(\ln 3) t=\ln (0.32) \\ & t=4.1486 \end{aligned}$ <br> ft on their $p$ and $k$ <br> 4.15 or awrt 4.1 | M1A1 <br> dM1 <br> A1 |




## Examiner reports

## Question 1

Question 1 was a familiar one to most candidates. It was generally well done by the majority of candidates although part (b) and finding answers in the range 0 to $2 \pi$ in part (c) did discriminate.

In part (a) most candidates were able to find $R$ and to make a worthwhile attempt at $\alpha$ usually via the tangent ratio. Degrees were occasionally used despite the range being given in radians. Some candidates were undecided and gave both degrees and radians, sometimes continuing with this throughout the question.
Part (b) was frequently incorrect with +25 as common as the correct answer of -25 . Another common answer was -1 and more surprisingly 0 . A less common error was to identify the value of $x$ for which the maximum/minimum would occur.

The majority of candidates attempted part (c) and realised the need to use the form found in part (a). There were therefore some very good solutions, in many of which the only error was to omit the second correct answer. Candidates should remember to derive additional values from their principal value before rearranging their equation. Not many gave all three values of $1.16,5.12$ and 7.44 for $(x+1.287)$. Rounding errors were common with 3.83 and 6.15 popular answers.

## Question 2

In part (a), the majority of candidates started with $\cos ^{2} \theta+\sin ^{2} \theta=1$ and divided all terms by $\cos ^{2} \theta$ and rearranged the resulting equation to give the correct result. A significant minority of candidates started with the RHS of $\sec ^{2} \theta-1$ to prove the LHS of $\tan ^{2} \theta$ by using both $\sec ^{2} \theta=\frac{1}{\cos ^{2} \theta}$ and $\sin ^{2} \theta=1-\cos ^{2} \theta$. There were a few candidates, however, who used more elaborate and less efficient methods to give the correct proof.

In part (b), most candidates used the result in part (a) to form and solve a quadratic equation in $\sec \theta$ and then proceeded to find $120^{\circ}$ or both correct angles. Some candidates in addition to correctly solving $\sec \theta=-2$ found extra solutions by attempting to solve $\sec \theta=\frac{2}{3}$, usually by proceeding to write $\cos \theta=\frac{2}{3}$, leading to one or two additional incorrect solutions. A significant minority of candidates, however, struggled or did not attempt to solve $\sec \theta=-2$.

A significant minority of candidates used $\tan \theta=\frac{\sin \theta}{\cos \theta}$ and $\sin ^{2} \theta=1-\cos ^{2} \theta$ to achieve both answers by a longer method but some of these candidates made errors in multiplying both sides of their equation by $\cos ^{2} \theta$.

## Question 3

This question was tackled with confidence by most candidates, many of whom gained at least 8 out of the 9 marks available.

In part (a), almost all candidates were able to obtain the correct value of $R$ although $3^{2}+5^{2}=36$ was a common error for a few candidates, as was the use of the "subtraction" form of Pythagoras. A minority of candidates used their value of $\alpha$ to find $R$. Some candidates incorrectly wrote $\tan \alpha$ as either $\frac{5}{3},-\frac{3}{5}$ or $-\frac{5}{3}$. In all of these cases, such
candidates lost the final accuracy mark for this part. A significant number of candidates found $\alpha$ in degrees, although many of them converted their answer into radians.
Many candidates who were successful in part (a) were usually able to make progress with part (b) and used a correct method to find the first angle. A significant minority of candidates struggled to apply a correct method in order to find their second angle. These candidates usually applied an incorrect method of ( $2 \pi$ - their 0.27 ) or ( $2 \pi$ - their $\alpha$ - their 0.27 ), rather than applying the correct method of ( $2 \pi-$ their principal angle - their $\alpha$ ). Premature rounding caused a significant number of candidates to lose at least 1 accuracy mark, notably with a solution of $0.28^{\text {c }}$ instead of $0.27^{\text {c }}$.

## Question 4

This type of question has been set quite frequently and the majority of candidates knew the method well. Most approached the question in the conventional way by expressing the fractions with the common denominator $(x-3)(x+1)$. This question can, however, be made simpler by cancelling down the first fraction by $(x+1)$, obtaining $\frac{2 x+2}{x^{2}-2 x-3}=\frac{2(x+1)}{(x-3)(x+1)}=\frac{2}{x-3}$. Those who used the commoner method often had difficulties with the numerator of the combined fraction, not recognising that $-x^{2}+1=1-x^{2}=(1-x)(1+x)$ can be used to simplify this fraction. If part (a) was completed correctly, part (b) was almost invariably correct. It was possible to gain full marks in part (b) from unsimplified fractions in part (a), but this was rarely achieved.

## Question 5

This question tested candidates on a 'real life' example of exponentials. Part (a) was very rarely incorrect. A few candidates did write $\mathrm{p}=2.5$ and then in (b) substituted 7.5 on the lefthand side. This seemed to be in an attempt to get straight to $\frac{1}{4} \ln 3$. A similar question to part (b) was asked in January and candidates had seemed to learn from that experience. Most seemed to score the first three marks. Some candidates still need to learn however that when an answer is given, it must be shown without doubt. For example justifying $\ln 3=-\ln \frac{1}{3}$ proved difficult for many. It was not uncommon to see $k=-\frac{1}{4} \ln 3$ going straight on to $k=\frac{1}{4} \ln 3$ without any explanation.

Of those who did provide an adequate proof (a large minority), it was common to see $\mathrm{e}^{-4 k}=\frac{1}{3} \Rightarrow \mathrm{e}^{4 k}=3$ used, and also $k=-\frac{1}{4} \ln 3 \Rightarrow k=\frac{1}{4} \ln \left(\frac{1}{3}\right)^{-1} \Rightarrow k=\frac{1}{4} \ln 3$. Less common was $k=-\frac{1}{4} \ln \left(\frac{1}{3}\right) \Rightarrow k=k=-\frac{1}{4}(\ln 1-\ln 3)$.

Part (c) was one of the more demanding parts of the paper; the derivative was not difficult but the numbers used made the question tricky. There were a pleasing number of completely correct solutions - by and large using the method shown on the mark scheme. A small number were able to proceed successfully with a change to powers of 3 . The latter method did cause a lot of problems for most who attempted it; many had $3 t^{-\frac{1}{4}}$ rather than the correct $3^{-\frac{1}{4} t}$. A common difficulty was processing the $-\frac{1}{4}(\ln 3) t$ and this was often caused by the lack of a
bracket around the $\ln 3$, so $\ln 3 t$ was sometimes processed in error. Most candidates went on to give a numerical answer, but it was possible to achieve the final mark for a correct exact answer.

## Question 6

This question was attempted by most candidates. The big difficulty here was in not using part (a) to help solve part (b). This is similar to what has happened in previous years and indicates that students are not aware that the question has been specifically designed to be solved in this way.
Most candidates coped well with the 'show that' in part (a). They were able to combine two fractions successfully and convert the expression from $\sin 2 \theta$ and $\cos 2 \theta$ into single angles, although the conversion of $\cos 2 \theta$ to a more appropriate form often required more than one stage of working. Those realising they should use $1-2 \sin ^{2} \theta$ for $\cos 2 \theta$ sometimes spoiled their proof by writing $1-1-2 \sin ^{2} \theta$ on the numerator thus reaching $-\frac{\sin \theta}{\cos \theta}$ and then conveniently 'losing' the - sign. Most of those reaching the final line gave sufficient evidence of method but a few left out crucial lines. It must be stressed that show that questions require all necessary steps.
Most candidates recognised the link between part (b)(i) and part (a) and used standard identities for $\sin 30^{\circ}$ and $\cos 30^{\circ}$, subsequently providing a correct proof using $\frac{\sqrt{3}}{2} \times \frac{1}{2}$ or equivalents explicitly on an intermediate line, followed by simplification to $2-\sqrt{3}$. Note that $\frac{1}{\sin 30}-\frac{\cos 30}{\sin 30}=2-\sqrt{3}$ gained only 1 mark as the intermediate step was missing. Some candidates followed the alternative path offered by expanding $\tan (45-30)$ or $\tan (60-45)$. This group usually failed to rationalise the denominator in the surd fraction $\frac{3-\sqrt{3}}{3+\sqrt{3}}$, gaining 2 of the 3 marks. There were also some instances of $\frac{\sin 15}{\cos 15}$ being used, with surd values used from the calculator gaining no credit at all. Similar unacceptable methods used $\tan (30-15)$ or $\tan (75-60)$.
Part (b)(ii) was done very simply by good candidates who were able to write down $\tan 2 x=1$ and the 4 correct values within a couple of lines. However, many others failed to recognise that the given equation was similar to the original expression in part (a), inevitably returning to first principles with many attempting the Pythagorean identity " $\cot ^{2} x=\operatorname{cosec}^{2} x-1$ ", often with little or no success.

## Question 7

Q7(a) was usually completed well and most candidates were able to score full marks. A few candidates found forming the single fraction challenging as they failed to recognise the lowest common denominator at the outset.
In Q7(b) the majority of candidates were able to use the quotient rule correctly and a number of candidates started by quoting the rule. A number of candidates used an incorrect form of the quotient rule, usually reversing the terms in the numerator. Some candidates failed to fully simplify their answer and a larger number who cancelled incorrectly which resulted in the final mark being lost. The common error seen was to change $-2 \times 2+10$ to $-2(\times 2+5)$. It
was also common to see responses where candidates misunderstood the notation and tried to find the inverse function. Some of these did however proceed to find $h^{\prime}(x)$ in Q 7 (c) and then went on to complete Q7(c) successfully.

Q7(c) was the most demanding part of this question. Those candidates who had cancelled incorrectly in Q7(b) found they had an unsolvable equation and tried to rearrange their equation in an attempt to form an equation that they could solve. Some candidates set their $\mathrm{h}^{\prime}(x)=0$ but then set the denominator of their derivative $=0$. A number of candidates failed to recognise that the maximum value of $\mathrm{h}(x)$ would be at the turning point and tried evaluating $\mathrm{h}(x)=0$ or $\mathrm{h}^{\prime}(0)$. It was common to see candidates forming inequalities for the range using their $x$ values instead of evaluating $\mathrm{h}(x)$. Of those with otherwise correct solutions, some lost the final mark by omitting the lower boundary for the range or by incorrectly using a strict inequality.

## Question 8

Part (a) was well answered, although candidates who gave the answer to 3 significant figures lost a mark.

In part (b) those candidates who realised that $x=15.3526 . .8 e^{-\frac{1}{8}}$ usually gained both marks, but a common misconception was to think that $10 e^{-\frac{1}{8}}$ should be added to the answer to part (a).

Part (c) proved a challenging final question, with usually only the very good candidates scoring all three marks. From those who tried to solve this in one stage it was more common to see $D=1$ or 10 or 20 or 13.549 , instead of than 15.3526 .., substituted into $x=D e^{-\frac{1}{8}}$. Many candidates split up the doses but this, unfortunately, often led to a complex expression in $T$, $3=10 e^{-\frac{T}{8}}+10 e^{-\frac{(T+5)}{8}}$, which only the very best candidates were able to solve. One mark was a common score for this part.

## Statistics for C3 Practice Paper Silver Level S4

| Qu | Max score | Modal score | $\begin{aligned} & \text { Mean } \\ & \% \\ & \hline \end{aligned}$ | Mean score for students achieving grade: |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | ALL | A* | A | B | C | D | E | U |
| 1 | 9 |  | 72 | 6.51 | 8.50 | 7.82 | 6.83 | 5.72 | 4.70 | 3.35 | 1.86 |
| 2 | 8 |  | 80 | 6.36 |  | 7.70 | 7.03 | 6.12 | 4.73 | 3.16 | 1.45 |
| 3 | 9 |  | 75 | 6.78 |  | 8.31 | 7.22 | 6.31 | 4.71 | 3.22 | 1.89 |
| 4 | 7 |  | 76 | 5.32 |  | 6.74 | 6.03 | 5.06 | 4.51 | 3.57 | 2.57 |
| 5 | 11 |  | 64 | 7.08 | 10.51 | 9.01 | 7.42 | 5.93 | 4.46 | 3.08 | 1.64 |
| 6 | 12 |  | 63 | 7.51 | 11.64 | 10.21 | 8.04 | 5.82 | 3.84 | 2.21 | 1.04 |
| 7 | 12 |  | 68 | 8.11 | 11.51 | 10.12 | 8.52 | 7.26 | 6.31 | 5.36 | 3.87 |
| 8 | 7 |  | 62 | 4.33 |  | 5.17 | 4.35 | 3.91 | 3.61 | 3.33 | 2.39 |
|  | 75 |  | 69 | 52.00 |  | 65.08 | 55.44 | 46.13 | 36.87 | 27.28 | 16.71 |

