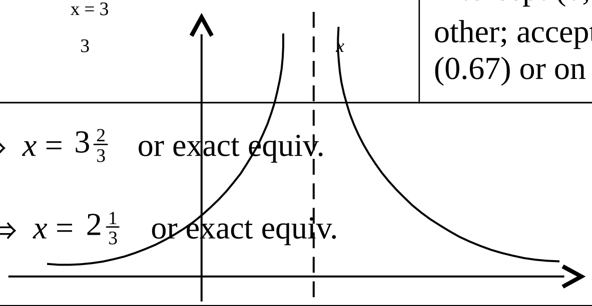
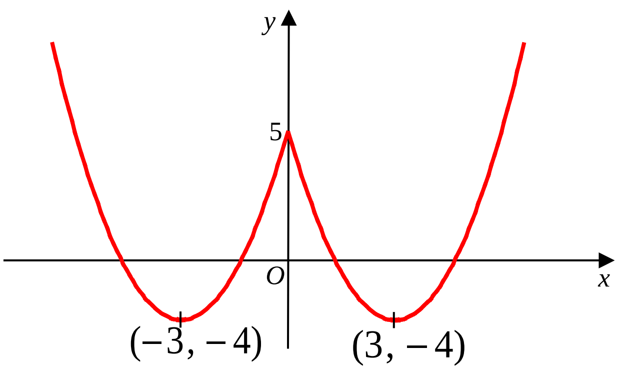
C3 re-sit paper, 9-10 February 2016: mark scheme

Question Number	Scheme	Marks
1. (a)	Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2$, $x_0 = 2.5$	
	$x_1 = \dfrac{2}{(2.5)^2} + 2$	M1
	$x_1 = 2.32$, $x_2 = 2.371581451...$	A1
	$x_3 = 2.355593575...$, $x_4 = 2.360436923...$	A1 cso (3)
(b)	Let $\mathrm{f}(x) = -x^3 + 2x^2 + 2 = 0$	
	$\mathrm{f}(2.3585) = 0.00583577...$	M1
	$\mathrm{f}(2.3595) = -0.00142286...$	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	A1 (3)
		[6]
2. (a)	$x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$	
	$\Rightarrow x^2(x+3) = 12 - 4x$	M1
	$\Rightarrow x^2 = \dfrac{12-4x}{(x+3)} \Rightarrow x = \sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1A1* (3)
(b)	$x_1 = 1.41$, *awrt* $x_2 = 1.20$ $x_3 = 1.31$	M1A1, A1 (3)
(c)	Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	$\mathrm{f}(1.2725) = (+)0.00827...$ $\mathrm{f}(1.2715) = -0.00821....$	M1
	Change of sign $\Rightarrow \alpha = 1.272$	A1 (3)
		[9]

Question Number	Scheme	Marks
3. (a)	f(0.75) = –0.18…	
	f(0.85) = 0.17…	M1
	Change of sign, hence root between x = 0.75 and x = 0.85	A1
		(2)
(b)	Sub x_0 = 0.8 into $x_{n+1} = \left[\arcsin\left(1-0.5x_n\right)\right]^{\frac{1}{2}}$ to obtain x_1	M1
	Awrt x_1 = 0.80219 and x_2 = 0.80133	A1
	Awrt x_3 = 0.80167	A1
		(3)
(c)	f(0.801565) = –2.7… × 10^{-5}	
	f(0.801575) = +8.6… × 10^{-6}	M1 A1
	Change of sign and conclusion	A1
		(3)
		[8]
4.	(a) $\mathrm{f}\left(2\right) = 0.38\ \ldots$	
	$\mathrm{f}\left(3\right) = -0.39\ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow$ root in $\left(2, 3\right)$ cso	A1 (2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$	M1
	$x_2 \approx 2.50498$	A1
	$x_3 \approx 2.50518$	A1 (3)
	(c) Selecting $\left[2.5045, 2.5055\right]$, or appropriate tighter range, and evaluating at both ends.	M1
	$\mathrm{f}\left(2.5045\right) \approx 6\times10^{-4}$	
	$\mathrm{f}\left(2.5055\right) \approx -2\times10^{-4}$	
	Change of sign (and continuity) $\Rightarrow$ root $\in \left(2.5045, 2.5055\right)$	
	$\Rightarrow$ root $= 2.505$ to 3 dp cso	A1 (2)
		[7]

Question Number	Scheme	Marks
5. (a)	$x^2-2x-3=(x-3)(x+1)$	B1
	$f(x)=\frac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \frac{2(x-1)}{(x-3)(x+1)}-\frac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\frac{x-3}{(x-3)(x+1)}=\frac{1}{x+1}$	A1 cso (4)
(b)	$\left(0,\frac{1}{4}\right)$ Accept $0<y<\frac{1}{4}$, $0<f(x)<\frac{1}{4}$ etc.	B1 B1 (2)
(c)	Let $y=f(x)$ $y=\frac{1}{x+1}$	
	$x=\frac{1}{y+1}$	
	$yx+x=1$	
	$y=\frac{1-x}{x}$ or $\frac{1}{x}-1$	M1 A1
	$f^{-1}(x)=\frac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\frac{1}{4}\right)$	B1 ft (3)
(d)	$fg(x)=\frac{1}{2x^2-3+1}$	
	$\frac{1}{2x^2-2}=\frac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3) **[12]**

Question Number	Scheme	Marks
6. (a)	Finding g(4) = k and f(k) = …. or fg(x) = $\ln\left(\frac{4}{x-3}-1\right)$	M1
	[f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] $=\ln 3$	A1 (2)
(b)	$y=\ln(2x-1)$ ⇒ $e^y=2x-1$ or $e^x=2y-1$	M1, A1
	$f^{-1}(x)=\frac{1}{2}(e^x+1)$ Allow $y=\frac{1}{2}(e^x+1)$	A1
	Domain $x\in\Re$ [Allow $\Re$, all reals, $(-\infty,\infty)$] independent	B1 (4)
(c)		
	Shape, and x-axis should appear to be asymptote	B1
	Equation x = 3 needed, may see in diagram (ignore others)	B1 ind.
	Intercept (0, $\frac{2}{3}$) no other; accept y = ⅔ (0.67) or on graph	B1 ind (3)
(d)	$\frac{2}{x-3}=3$ ⇒ $x=3\frac{2}{3}$ or exact equiv.	B1
	$\frac{2}{x-3}=-3$, ⇒ $x=2\frac{1}{3}$ or exact equiv.	M1, A1 (3)
		[12]
7. (a)(i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1 (4)
(b)		B1 B1 B1

Question Number	Scheme	Marks
		(3)
(c)	$\mathrm{f}(x) = (x-3)^2 - 4$ or $\mathrm{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1)
		[10]

Question Number	Scheme	Marks
8. (a)	(i) $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{\mathrm{d}}{\mathrm{d}x}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x) \times \frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{3}{3x}$	M1A1
		(3)
	(ii)	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-1)^5 \times -10 - (1-10x) \times 5(2x-1)^4 \times 2}{(2x-1)^{10}}$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{80x}{(2x-1)^6}$	A1
		(3)
(b)	$x = 3\tan 2y \quad \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1 + \tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6(1+(\frac{x}{3})^2)} = \left(\frac{3}{18+2x^2}\right)$	M1A1
		(5)
		[11]

(This is the Edexcel "Silver Two" paper)